C and the $n^2 \times n^2$ matrix D. Writing (1) as

$$[P_{k+1}] = [P_k] + \{I \times (A - GP_k)' + (A - GP_k)' \times I\}^{-1} \cdot [-P_k(A - GP_k) - (A - GP_k)'P_k - P_kGP_k - Q] \quad (2)$$

and regrouping terms in (2) gives

$$[P_{k+1}] = \{I \times (A - GP_k)' + (A - GP_k)' \times I\}^{-1} \cdot [-P_kGP_k - Q] + [P_k] - \{I \times (A - GP_k)' + (A - GP_k)' \times I\}^{-1}[P_k(A - GP_k) + (A - GP_k)'P_k]] \quad (3)$$

With the use of the Kronecker product, it can be shown that

$$[P_k(A - GP_k) + (A - GP_k)'P_k] = \{I \times (A - GP_k)' + (A - GP_k)' \times I\}[P_k] \quad (4)$$

Substituting (4) in (3) yields

$$[P_{k+1}] = \{I \times (A - GP_k)' + (A - GP_k)' \times I\}^{-1} \cdot [-P_kGP_k - Q] \quad (5)$$

In view of (4), (5) can be further simplified to give the desired iterative scheme

$$P_{k+1}(A - GP_k) + (A - GP_k)'P_{k+1} = -P_kGP_k - Q \quad (6)$$

The modified iterative scheme requires at each iteration only to solve a matrix Lyapunov equation of the type

$$SF + F'S = -T \quad (7)$$

An algorithm that requires little computer storage space and is fast for solving (7) is as follows[2]:

$$S_1 = (h/3)(4\Phi'T\Phi + 2T)$$

$$S_{i+1} = (\Phi')^{2i}S_i(\Phi)^{2i} + S_i \qquad i = 1,2,3, \ldots$$

$$S = \lim_{i\to\infty} S_i - \frac{h}{3} T$$

where

$$\Phi = [I - (h/2)F + (h/12)F^2]^{-1}[I + (h/2)F + (h/12)F^2]$$

and h is the discrete time interval. This algorithm requires $4n^2$ words of computer memory and converges to 6 significant figure accuracy typically in $30n^3$ time units.† For example, for a 50th-order system, the computer storage space and the computer time required are $10K$ words and 3.75×10^6 time units per iteration, respectively, whereas the corresponding figures for Blackburn's scheme are $1636K$ words and 6.9×10^8 time units per iteration.‡

It is interesting to note that (6) is identical to the algorithm reported by Puri et al.[3] and Kleinman.[4] However, it appears that their method of solution is based on (5) and consequently suffers the same limitations as (1).

† One time unit is equivalent to the time (in microseconds) required to perform one multiplication and one addition in a digital computer.

‡ Time estimation is based on the fact that Gaussian elimination is a $\frac{1}{3}r^3$ time unit process, where r is the number of equations to be solved.

References

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Errata: "Thermally Induced Membrane Stress in a Circular Elastic Shell"

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In the third line of Eq. (15), move the closing bracket to the right, to follow $(\tau - x)$. In the last line of that equation, $\tau - x$ should be enclosed in parentheses. In the second line of Eq. (18), the 8 should be changed to 2 in the numerator of the fraction following the summation sign.

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Announcement: 1968 Author and Subject Indexes

It has been the custom to publish the annual author and subject indexes of the AIAA journals in the last issue of the year. This year, however, with the approval of the Publications Committee, we will publish a combined index of the four journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft, and Journal of Hydronautics*). All topic headings will be included, whether or not anything on that subject was published. The index will be mailed to all subscribers to the journals in January 1969. We hope that readers will find the combined index more convenient to use than four separate ones.

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